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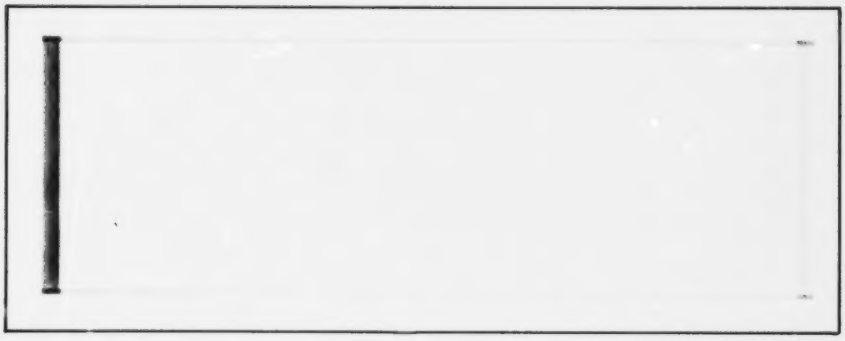

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A MULTI-OUTPUT MODEL OF THE CANADIAN COW-CALF INDUSTRY

(Working Paper 16/86)

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Abstract:

The purpose of this paper is two-fold. First, it is to specify and estimate a multi-output, multi-input aggregate profit function for the cow-calf industry in Western Canada from which a number of summary statistics of output and input flexibility, including the short-run elasticities of output supply and input demand and measures of substitutability of outputs and of inputs are derived. Second, it is to illustrate that a Cobb-Douglas (C-D) functional form can not be used in estimating multi-output, multi-input technologies because a C-D specification fails to satisfy the convexity conditions. Hence a dual relationship will not exist between the transformation function and the profit functions (Diewert 1973 and 1974). This means that policy simulations and forecasts exploiting C-D specifications and other specifications which fail to satisfy the convexity conditions may well be in error.

Two conclusions can be drawn from our results. First, cow calf producers respond relatively more to changes in current prices than to expected future price changes. This implies that short term price fluctuations in the cattle market will result in significant changes in current cattle supply. Second, there is significant scope for changing the output composition between cattle and crops on cow-calf farms. This

suggests that during periods of falling cattle prices, cow-calf producers will shift away from cattle production and increase the production of crops.

Keywords:

Multi-output aggregate profit function, convexity conditions, transformation function, cattle cycle, Hessian, principle minors, flexible functional form (translog).

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Introduction

The purpose of this paper is two-fold. First, it is to specify and estimate a multi-output, multi-input aggregate profit function for the cow-calf industry in western Canada from which a number of summary statistics of output and input flexibility, including the short run elasticities of output supply and input demand and measures of substitutability of outputs and of inputs, are derived. Second, it is to illustrate that a Cobb-Douglas (C-D) functional form can not be used in estimating multi-output, multi-input technologies because a C-D specification fails to satisfy the convexity conditions. Hence a dual relationship will not exist between the transformation function and the profit functions (Diewert 1973 and 1974).

The cattle industry is an important sector of agricultural production in Canada. For the period 1970 to 1982, farm cash receipts from the sale of cattle and calves varied between 18.4% and 34.9% of total farm cash receipts, second only to the grains (wheat, barley, etc.) industry.[1] Additionally, international trade in beef animals and meat products has increased significantly in recent years. Between 1977 and 1982 the value of Canadian imports of live animals and meat products increased from $357 million to $1075 million. The corresponding figures for imports rose from $325 million to $402 million.[2]

The U.S. is Canada's largest trading partner in beef products accounting for approximately 90% of total exports of cattle and calves and 85% of total dressed beef and veal.

Canada produces approximately 10% of total North American beef

output. Because of the size and proximity of the U.S. market and the virtual free trade in beef cattle between the two countries, Canadian beef prices are closely relatd to U.S. beef prices. Figure 1 illustrates this relationship with quarterly choice steer prices in Canadian funds, in Calgary and Omaha, for the period 1977 to 1982. If changes in supply or demand conditions for beef in Canada result in Canadian prices becoming significantly higher than U.S. beef prices, arbitrage will result in U.S. cattle entering the Canadian market and consequently decreasing Canadian beef prices. One would expect that U.S. cattle would be imported to Canada if the Canadian beef price is greater than the U.S. price plus transportation costs to Canadian markets plus transaction costs, which include a small import tariff.

One distinctive feature of the cattle industry is large variations over time in cattle production. This has become known as the "cattle cycle". The cattle cycle is represented in Figure 2 by the inventories of cows and heifers on farms in Canada for the period 1950-82. Female inventories were at a cyclical low point in 1950: they reached a peak in 1965, a relative low point in 1968, and again peaked in 1975. Included in Figure 2 is a curve representing the ratio of choice steer prices to the price of feed barley over the period 1950-82. Generally, the price ratio is the reciprocal of inventory movements, but prices precede the turning points of inventories by several years. The response lag of inventories following changes in prices reflects the biological lag between when cow-calf farmers make production plans and when such plans are reflected in herd size.

FIGURE 1

Choice Steer Prices, in Canadian Funds, Calgary and Omaha

Source: Agriculture Canada, Canadian Livestock and Meat Trade Report, Ottawa, Queen's Printer, annual.

Figure 2

Sources: Statistics Canada, Livestock and Animal Product Statistics, Cat. No. 23-203, Ottawa, Queen's Printer, annual.

Statistics Canada, Handbook of Agricultural Statistics, Cat. No. 21-516, Part 1, Ottawa, Queen's Printer, annual.

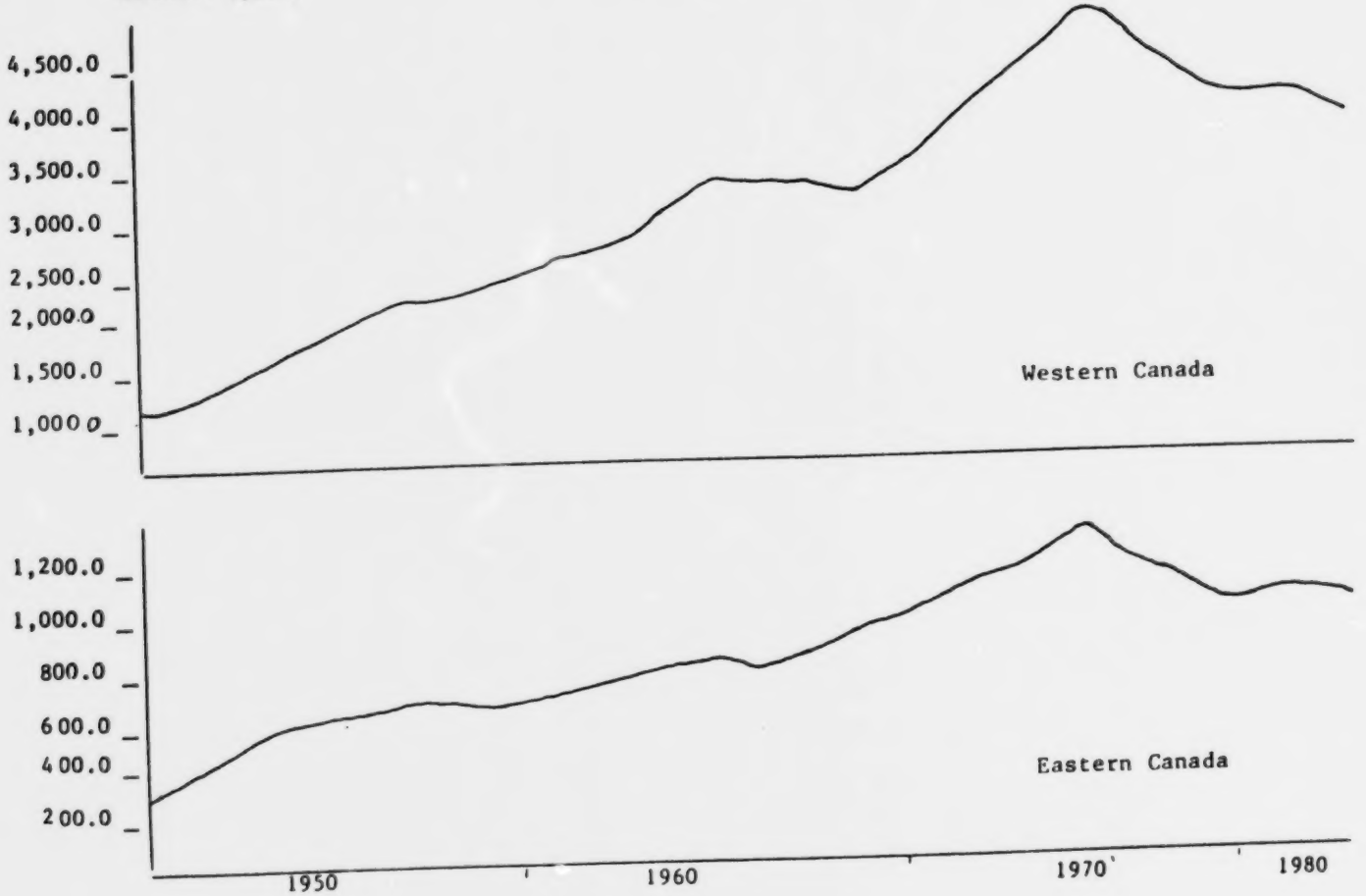
A better description of the cattle cycle is gained from examining changes in female inventories in the beef herds in western Canada compared to herds in eastern Canada. Figure 3 shows changes in female inventories for the two regions for the period 1950-1982. Although there are some cyclical trends in the eastern beef herd, the more pronounced variations occur in the western herd. This reflects the fact that 80 percent of the Canadian beef breeding herd is in western Canada. Consequently, changes in Canadian beef production are due primarily to inventory changes in the beef herd in western Canada.

To gain some insight into the causes of the cattle cycle, consider Figure 4 which illustrates steer and female slaughter for the period 1960-82. Steer slaughter is more stable than female slaughter. This indicates that as the herd expanded (say between 1970 and 1975), female animals were held back from the market and retained in the breeding herd to produce new animals. Consequently, during this period of the cycle, female slaughter is less than steer slaughter. However, the decline in the herd after 1975 coincides with farmers reducing their breeding herds (culling cows and slaughtering heifers) resulting in a large female slaughter.

The positions of the industry along a cycle can generally be identified by the slaughter of females as a ratio of steer slaughter. Figure 5 shows this ratio for the period 1960-82. During an expansionary phase of the cycle, for example between 1970 and 1975, female slaughter is significantly less than steer slaughter whereas during a contractionary phase of the cycle, after 1975, female slaughter is

Figure 3

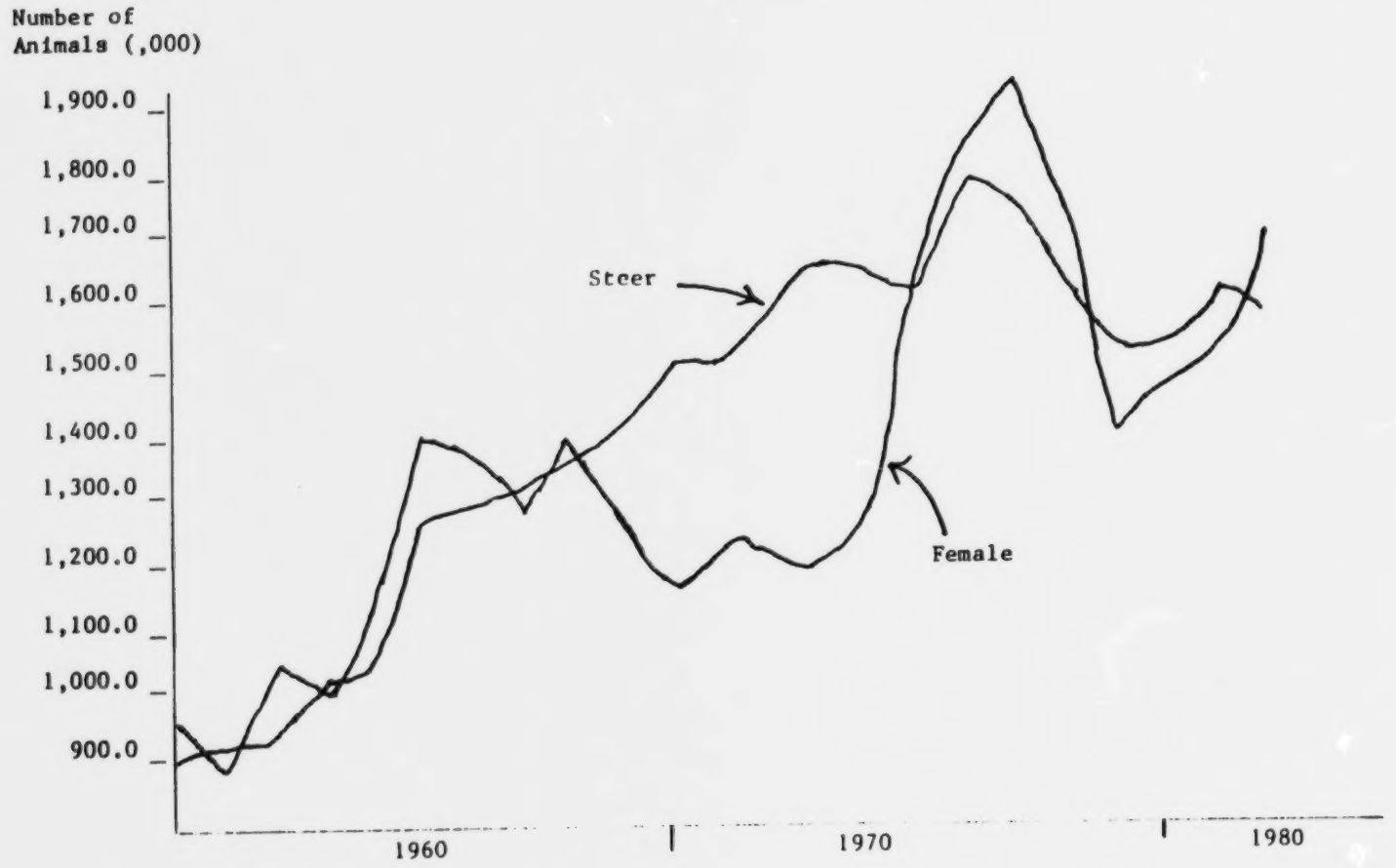
Inventories of Cows and Heifers on Farms, Western and Eastern Canada



Source: Statistics Canada, Livestock and Animal Product Statistics, Cat. No. 23-203, Ottawa, Queen's Printer. annual.

Figure 4

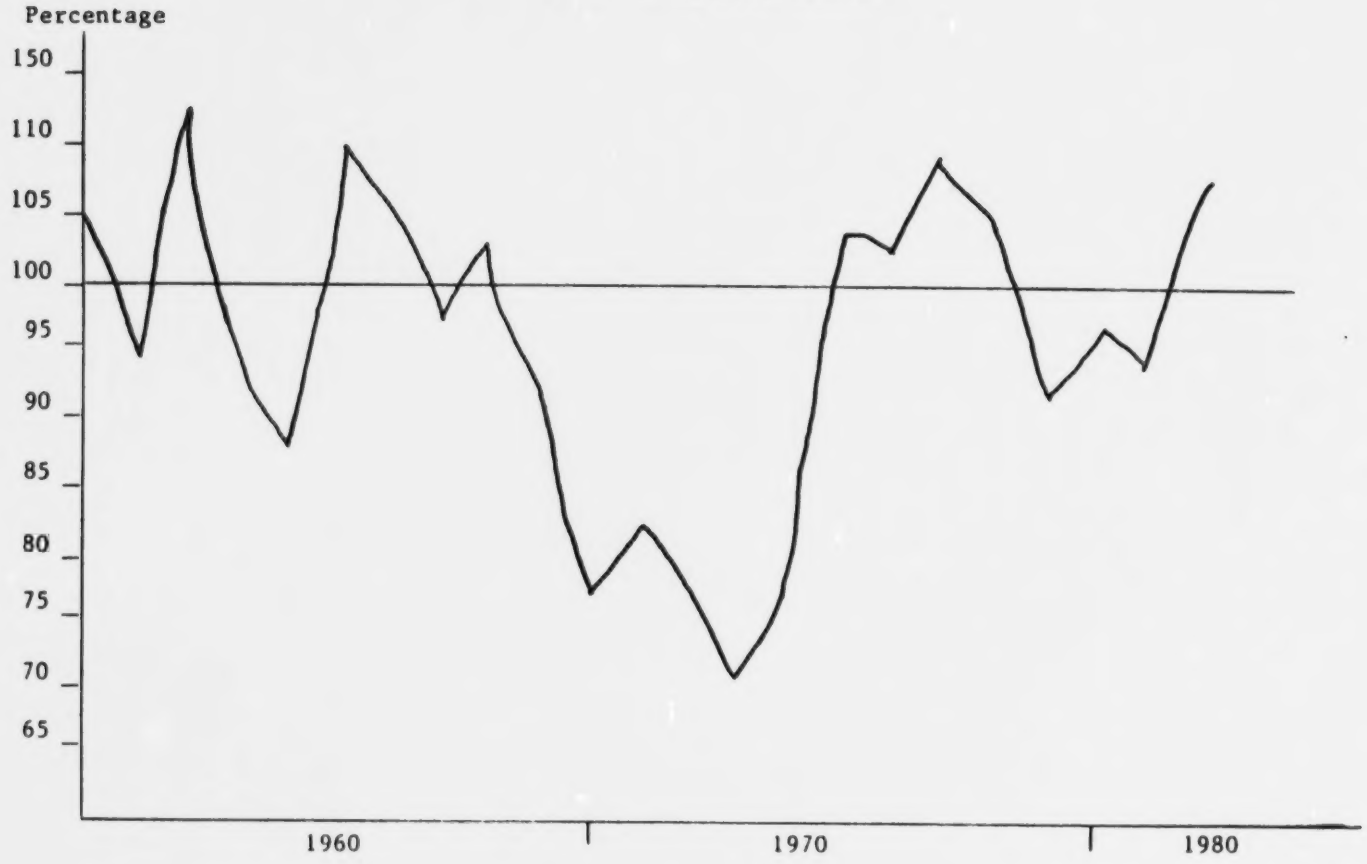
Steer and Female Slaughter, Canada



Source: Statistics Canada, Livestock and Animal Product Statistics, Cat. No. 23-203, Ottawa, Queen's Printer, annual.

Figure 5

Female Slaughter as a Percentage of
Steer Slaughter, Canada



Source: Statistics Canada, Livestock and Animal Product Statistics, Cat. No. 23-203, Ottawa, Queen's Printer, annual.

significantly greater than steer slaughter. A ratio of female to steer slaughter of less than one indicates an expansionary phase. Conversely, a ratio of more than one indicates a contractionary phase.

Both Figure 4 and 5 suggest that there is a significant relationship between the cattle cycle and the slaughter or retention of female animals. Marshall (1964) describes this relationship as follows:

> "When the price of cattle is high relative to other production possibilities the tendency is to hold back cows and heifers for breeding. Inventories are thus augmented, marketings reduced and prices strengthened. As inventory numbers build up and the progeny of increased cow numbers reach market weight marketings increase. Eventually increased marketings reduce prices to a point that discourages further expansion and eventually some liquidation of inventories takes place. The following decline in marketings results in prices increasing and the beginning of a new cycle."

Cow-calf producers have always had to adjust and respond to the cattle cycle as part of the biological and economic environment in which they operate. Before attempting to estimate the structural parameters of the cow-calf industry we will first show that the C-D functional form is not appropriate for modelling multi-output technologies.

Convexity Restrictions in the Multi-Output C-D Profit Function

Consider the maximization problem. The objective of the multi-output, multi-input firm is to maximize the difference between total revenue and total cost subject to a concave transformation

function, $F(q,C,Y) = 0$, where q is an I-dimensional vector of inputs [$q = (q_1,\ldots, q_I)$], C is an M-dimensional vector of fixed factors, and Y is an N-dimensional vector of output supply. The transformation function F(.) shows the technological possibilities available for combining the vector of inputs with the vector of fixed factors to determine the vector of output supply. F(.) is increasing in the individual components of the vectors q and C, indicating that the marginal products of inputs and fixed factors are strictly positive.

This maximization problem can be written as:

$$(1) \quad \Pi(P,W;C) = \max_{Y,q} [P^TY - W^Tq: (q,C,Y) \in F],$$

where P is a vector of output prices and W is a vector of input prices. $\Pi(\cdot)$ is defined as the variable profit function[3] dual to $F(\cdot)$ and is a function of output prices, input prices, and any fixed factors (Lau 1972).

In order for there to exist a dual relationship between the production possibility set and the variable profit function, certain regularity conditions must be satisfied by each function (Diewert 1973 and 1974). Generally, the regularity conditions on $F(\cdot)$ ensure that the technology exhibits non-increasing marginal rates of transformation, free disposal, and is well-behaved in the sense that a bounded vector of inputs can produce only a bounded vector of outputs. The regularity conditions on $\Pi(\cdot)$ ensure that the profit function satisfies monotonicity, convexity in prices, linear homogeneity, and that $\Pi(.)$ is non-negative.

Diewert (1973) proves that given any function $\Pi(\cdot)$ satisfying the regularity conditions, there exists a unique $F(\cdot)$ which satisfies the regularity conditions on the technology and which generates $\Pi(\cdot)$ through Equation (1). Thus, $F(\cdot)$ and $\Pi(\cdot)$ are equivalent representations of the technology and therefore $\Pi(\cdot)$ may be used to characterize the technology and to test for structure.

The convexity properties of the multi-output C-D profit function can now be determined. Assume that the transformation function $F(\cdot)$ in Equation (1) is C-D in structure. The C-D functional form has the interesting characteristic of being self-dual (i.e., the variable profit function will also be C-D in structure (Lau and Yotopoulos)). To anticipate the empirical application, the three-output, three-input, one fixed factor variable C-D profit function can be written as:

$$(2) \quad \Pi(P,W;C) = AP_1^{\alpha_1} P_2^{\alpha_2} P_3^{\alpha_3} W_1^{\beta_1} W_2^{\beta_2} W_3^{\beta_3} C^{\gamma} \quad .$$

Equation (2) must satisfy the regularity conditions and in particular, it must be convex in prices.

Convexity requires that all principle minors of the Hessian matrix formed from the second order partial derivatives of the profit function with respect to prices be non-negative. This Hessian can be

written as follows:

$$H = \left[\begin{array}{ccc|ccc} \Pi_{P_1P_1} & \Pi_{P_1P_2} & \Pi_{P_1P_3} & \Pi_{P_1w_1} & \Pi_{P_1w_2} & \Pi_{P_1w_3} \\ \Pi_{P_2P_1} & \Pi_{P_2P_2} & \Pi_{P_2P_3} & \Pi_{P_2w_1} & \Pi_{P_2w_2} & \Pi_{P_2w_3} \\ \Pi_{P_3P_1} & \Pi_{P_3P_2} & \Pi_{P_3P_3} & \Pi_{P_3w_1} & \Pi_{P_3w_2} & \Pi_{P_3w_3} \\ \hline \Pi_{w_1P_1} & \Pi_{w_1P_2} & \Pi_{w_1P_3} & \Pi_{w_1w_1} & \Pi_{w_1w_2} & \Pi_{w_1w_3} \\ \Pi_{w_2P_1} & \Pi_{w_2P_2} & \Pi_{w_2P_3} & \Pi_{w_2w_1} & \Pi_{w_2w_2} & \Pi_{w_2w_3} \\ \Pi_{w_3P_1} & \Pi_{w_3P_2} & \Pi_{w_3P_3} & \Pi_{w_3w_1} & \Pi_{w_3w_2} & \Pi_{w_3w_3} \end{array}\right]$$

A B D C

where double subscripts indicate second order derivatives. For convenience of exposition the Hessian is partitioned into four sub-matrices. A necessary condition for the Hessian to be convex is that the principle minors of sub-matrix A must be non-negative (i.e., the Hessian must be convex in output prices). Similarly, because the order in which prices appear in the Hessian is arbitrary, the principle minors of sub-matrix C must also be non-negative (i.e., the Hessian must be convex in input prices). If it can be determined that either sub-matrix A or C is non-convex then H is non-convex. The question then is what are the necessary and sufficient conditions to ensure convexity in output and input prices?

On the input side, a necessary condition for convexity requires that the second order partial derivatives of Equation (2) with respect to input prices are non-negative or:

$$(3) \quad \Pi_{w_jw_j} = \beta_j (\beta_j - 1) \frac{\Pi}{w_j^2} \geq 0 \qquad j = 1, 2, 3.$$

Given that Π must be non-negative, the satisfaction of Equation (3)

requires $\beta_j \leq 0$ for all j. This restriction, however, is ensured by the monotonicity condition. In addition, $\beta_j \leq 0$ for all j also ensures the satisfaction of the sufficient condition for convexity in input prices (i.e., the principle minors of sub-matrix C are non-negative.) Consequently, the condition that derived demand curves have non-positive slopes is both a necessary and a sufficient condition for convexity in input prices.

One might expect that a similar restriction on the output side would ensure convexity in output prices. However, the satisfaction of the monotonicity condition is neither a necessary nor a sufficient condition for convexity. To illustrate this, a necessary condition for convexity requires that the second order partial derivatives of Equation (2) with respect to output prices are non-negative or:

$$(4) \quad \Pi_{p_i p_i} = \alpha_i(\alpha_i - 1)\frac{\Pi}{p_i^2} \geq 0 \qquad i = 1, 2, 3.$$

The sign of $\Pi_{p_i p_i}$ will depend on the parameter α_i. Monotonicity requires $\alpha_i \geq 0$ for all i but the satisfaction of Equation (4) requires $\alpha_i \geq 1$ for all i. Consequently, the condition that output supply curves have non-negative slopes does not ensure convexity in output prices.

Keeping in mind that the α_i's are defined as the revenue share of output i in total profit, the convexity restrictions require that the revenue received for each output i must be greater than the total profit received by the firm. Clearly, in the multi-output case, there is no prior reason to expect these restrictions to be satisfied. However, if

by chance the necessary conditions are satisfied (i.e., $\alpha_i \geq 1$ for all i) this will preclude the satisfaction of the sufficient conditions. To illustrate this it is only required to show that at least one principle minor of the Hessian is negative for $\alpha_i \geq 1$. Consider the second principle minor (H_2):

$$|H_2| = \begin{vmatrix} \alpha_1(\alpha_1-1)\frac{\Pi}{p_1^2} & \frac{\alpha_1\alpha_2\,\Pi}{p_1p_2} \\ \frac{\alpha_2\alpha_1\,\Pi}{p_1p_2} & \frac{\alpha_2(\alpha_2-1)\,\Pi}{p_2^2} \end{vmatrix} = \alpha_1\alpha_2 - \alpha_1^2\,\alpha_2 - \alpha_1\alpha_2^2$$

For $\alpha_i \geq 1$ this determinate will be negative and convexity can not be satisfied. Consequently, the theoretical restrictions required to ensure a duality between the profit and transformation functions can not be achieved and therefore the C-D functional form is not appropriate for modelling multi-output technologies.

At this point one might be willing to completely abandon the C-D functional form and choose instead a flexible functional form (FFF) to represent the multi-output profit function in which case the FFF chosen must also satisfy the regularity conditions. However, with the increased application of FFF it is apparent that the estimated functions generally fail to satisfy some of the theoretical conditions (Diewert and Wales). This failure is not due to restrictions imposed by the FFF but rather to the quality of the data used in the analysis.[4] Consequently, in modelling technologies the researcher is faced with the choice of specifying either a FFF, which theoretically will allow for testing economic restrictions, or a more restrictive functional form that imposes economic constraints

on the technology. The decision will depend on the problem at hand and the quality of data available for analysis.

In agricultural markets it is not uncommon to have available good quality data on output variables whereas input data, obtained from estimates provided by producers, are less accurate. Given such data characteristics an alternative modelling procedure is to model the output structure using a FFF but to maintain a more restrictive functional form over input variables. To proceed in this fashion one must assume that the technology is weakly separable in output variables.[5] This implies that the profit function will be weakly separable in output prices and can be written as

$$\Pi(P_I(P);\ W;\ C) \quad ,$$

where $P_I(P)$ is defined as an aggregator function or aggregate price index. The separability assumption also implies a two-stage maximization procedure: optimize the output mix within the aggregator function and then optimize over aggregate output and variable inputs. This requires specifying functional forms for both $P_I(P)$ and $\Pi(.)$. For the aggregate price index a translog functional form is postulated. The translog does not require restrictions on revenue shares to satisfy convexity and, as Fuss has shown, is a discrete approximation to the Divisia price index. A C-D specification is maintained over the aggregate profit function.

The translog price index for the three-output case can be written as follows:[6]

$$(5) \quad \ell nP_I = \ell n\alpha_o + \sum_{i=1}^{3} \alpha_i \ell nP_i + \tfrac{1}{2} \sum_{i=1}^{3} \sum_{k=1}^{3} \alpha_{ik} \ell nP_i \ell nP_k \quad ,$$

where n is the log operator, P_I is the aggregate price index and P_i is the ith output price.

To satisfy the regularity conditions, Equation (5) must be convex in prices. This can be checked by calculating the eigenvalues for the corresponding Hessian matrix -- where convexity requires that each eigenvalue must be greater than, or equal to, zero.

After estimating the parameters in Equation (5), an aggregate price index can be calculated and used to rewrite Equation (2) as a single output normalized C-D profit function (Lau 1976):

$$(6) \qquad \Pi^* = A W_1^{*\beta_1} W_2^{*\beta_2} W_3^{*\beta_3} C^{\gamma} ,$$

where input prices and profit are normalized by P_I. Equation (6) will now satisfy all convexity requirements.

The estimation of Equation (6) will provide input elasticities and other characteristics of the cost structure of the firm whereas output elasticities and other characteristics of the output structure can be calculated from Equation (5). The procedures described above will be used in the next section to model and to estimate the structure of the cow-calf industry in western Canada.

A C-D Model of the Cow-Calf Industry

The cow-calf farmer is engaged in the primary activity of reproducing animals and selling the progeny. A secondary activity is the selling of cull cows (and bulls). These activities are distinct from the specialized feeder operator whose primary role is the production of finished beef. The basic decision of the cow-calf farmer is whether to sell a calf now or feed to heavier weights before selling. This decision will depend on the prevailing and expected economic conditions: the price of animals at different weights; the availability of pasture and its quality; the price of associated inputs; and the opportunity cost of the farmer. At any point in time, therefore, it is likely that a cow-calf farmer will have a variety of animals in his herd (eg., bulls, cows, steers, heifers, calves) at different weights and ages.

In managing the herd, the cow-calf farmer is faced with three major decisions (Yver 1971):

a. determining optimal herd size (and associated optimal input levels);
b. determining the optimal numbers of different types of animals in the herd; and
c. determining whether an animal should be sold or retained in the herd for the purpose of producing more animals.

The first and third decisions are typical production decisions (i.e., determining the size of plant and output rate) while the second is basically a portfolio decision.

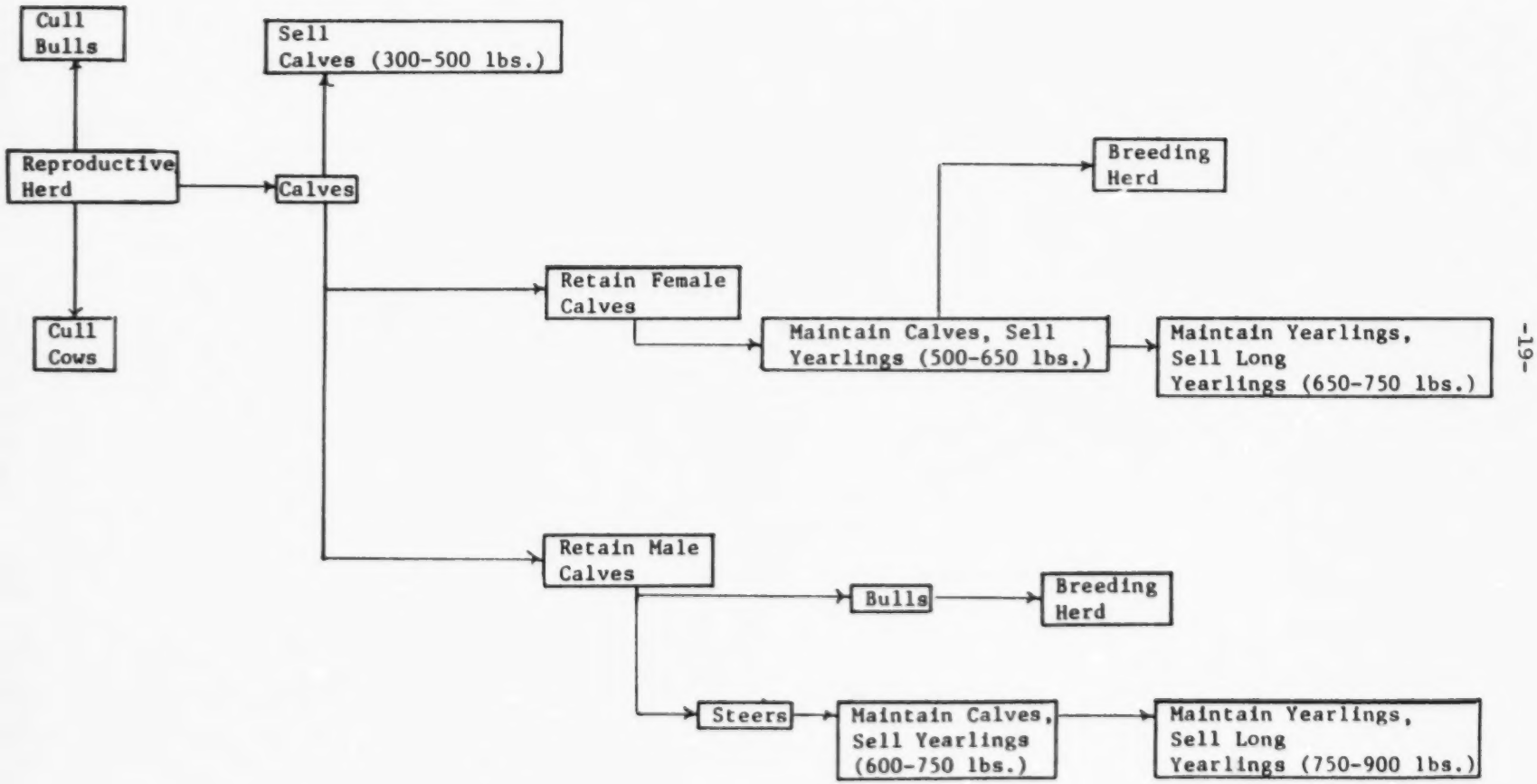
Figure 6 helps to describe the economic options available to the cow-calf producer. The focus of a cow-calf farmer's management decisions is the reproductive herd which includes bulls, cows and heifers. The number of calves produced in any one year depends on the number of cows and heifers bred nine months earlier. A successful calving rate of 85% is considered average. Generally, because of Canadian weather conditions, calving is timed to take place in the early spring. The new calves can be sold to feedlots in the fall of the year in which they are born or retained in the herd over the winter. The cow-calf farmer has available a number of production alternatives for the retained calves. Bull calves can be retained for breeding within the reproductive herd or they can become steers. Steers can be sold to feedlots as yearlings at approximately 600-750 pounds or maintained on pasture and sold as long yearlings at appoximately 750-900 pounds (feedlots sell finished steers at approximately 1000-1100 pounds).

Female calves can be kept as replacement heifers or sold to feedlots. Heifers can be sold as yearlings at approximately 500-650 pounds or maintained on pasture and sold as long yearlings at approximately 650-750 pounds. (Feedlots sell finished heifers at approximately 850-950 pounds). Of course, the decision on whether to use a heifer as a replacement can be made up to the time the animal is sold.

In the case of steers, the decision of the farmer is quite straightforward: he must decide on the optimal weight and time to sell the animal. In the case of bulls, cows, and heifers, the decision is more complicated. He must decide whether to sell the animal, retain it

Figure 6

Flow Diagram of Cattle Production Decisions



for further fattening (heifers), or incorporate it into the breeding herd for producing calves.[7]

Assume that the objective of the cow-calf farmer is to maximize the short-run market value of his wealth. In modelling this maximizing problem, the farmer is assumed to have a given stock of animals at the beginning of the period. Moreover, the farmer knows (with certainty) the prevailing output prices and input prices. But the farmer does not know cattle prices next period. Rather, he forms some expectation of what cattle prices will be next period. The farmer combines the beginning stock of animals with a vector of variable inputs. Then, responding to an economic environment with known output prices, input prices, and his expectations about cattle prices next period, the farmer determines the stock of animals retained at the end of the period and output supply during the period. It is important to note that current output supply is valued at prevailing output prices but animals retained at the end of the period are valued at their expected price next period discounted to the present or:

$$P_i^e = \frac{1}{(1 + r)} E(\bar{P}_i^e) \quad ,$$

where P_i^e is the discounted expected cattle price for each animal category i, r is the discount rate, and $E(\bar{P}_i^e)$ is the expected cattle price for each animal category i.

This maximizing problem can be written in terms of Equation (1) as (Diewert 1972):

$$(7) \qquad \Pi(P,W,P^e; C^b) = \max_{Y,q,C^e} [P^T Y - W^T q + P^{eT} C^e : (q,C^b,C^e,Y) \in F] \quad ,$$

where Y is a vector of current output supply, C^e is a vector of animals retained at the end of the period, P^e is a vector of expected cattle prices next period, C^b is the beginning stock of cattle, and all other variables are as previously defined. By assuming weak separability in output variables the profit function in Equation (7) can be rewritten as:

$$(8) \qquad \Pi(P_I(P,P^e),W;C^b).$$

The variable profit function $\Pi(.)$ is now a function of current output prices, expected cattle prices, current input prices, and the beginning stock of cattle. An econometric model can be postulated using Equation (8) by specifying functional forms for $P_I(.)$ and $\Pi(.)$, by determining price expectations of cattle producers, and by specifying a stochastic disturbance term for each equation.

It will be assumed that cow-calf farmers' expectations of future cattle prices can be represented exactly by the prediction of a polynomial distributed lag model of past annual own-prices (Almon). Average annual prices, for the period 1946 to 1983, for five major auction markets in western Canada (Calgary, Edmonton, Regina, Saskatoon and Winnipeg) were provided by Agriculture Canada and used to estimate an Almon lag prediction equation for each animal category in each market location. In order to generate the best fit to the data alternative specifications of the model were attempted. The final model was selected on the basis of R^2-values and t-statistics for the estimated coefficients. A polynomial of degree three with a lag length of four provided the best fit to the data.

From the estimated equations, a prediction is generated for each animal category, for each year, to correspond with the main data base. For steers and calves, the predicted Almon price is discounted back one period and defined as the expected price for each animal. For cows and heifers, however, it is assumed that farmers value a female animal next period at its own expected price plus the price expected from the sale of its newborn calf. This measure is discounted back one period and defined as the expected price for each female animal.

The main data base used in estimating the econometric model was assembled from the Farm Expenditure Survey (FES) (for inventory and expenditure data on cow-calf farms), Cansim (for prices of farm inputs), and Livestock Market Review (for cattle output prices).

For purposes of extracting information on cattle production from the FES, a cow-calf farm is defined to have thirty or more beef-breeding cows in inventory. According to the definition, Statistics Canada provided annual inventory and expenditure data for cow-calf farms in fourteen soil zone locations in western Canada for the period 1978 to 1981.

For econometric specification, it was decided to postulate three aggregate output groups, three aggregate input groups, and one fixed stock variable for the cow-calf farm. The output variables were defined as: (i) total cattle supply sold off farms; (ii) total end-of-period cattle inventories; and (iii) total crop supply.[8] The input variables were defined as (i) labour; (ii) capital; and (iii) materials and services.[9] Finally, the one fixed factor is defined as the total

beginning stock of cattle. Profit is defined as the combined total value of the three outputs minus the total cost of the variable inputs.

The cross-sectional, time-series nature of the data sample utilized in the econometric estimation necessitates the use of covariance estimators to take account of yearly differences in the data (Judge, et al) in the form of yearly dummy variables (Di) attached to the constant term.

In specifying a stochastic disturbance term, it is assumed that any deviation from the profit-maximizing level is due to random errors in optimization. This random disburbance is modelled by appending an additive disturbance term to each equation.

The translog price index can not be estimated directly because the aggregate price index (P_I) is unknown. However, the coefficients of the equation can be estimated from the revenue share equations which are obtained by applying Hotelling's lemma to Equation (5). The share equations can be written incorporating yearly dummy variables and random error terms as:

$$(9) \quad S_i = \alpha_i + \alpha_{i1} \ln P_1 + \alpha_{i2} \ln P_2^e + \alpha_{i3} \ln P_3 + \sum_{j=1}^{3} \gamma_j D_j + \xi_i \qquad i = 1,2,3.$$

where S_i is the revenue share of output i in total revenue, P_1 is the cattle output price index, P_2^e is the expected cattle price index, P_3 is the crop output price index, D_j are yearly dummy variables (base year 1978), and ξ_i is a random error term.

Econometric estimation of Equation (9) will generate estimates of all coefficients in Equation (5) except the intercept term, $\ln\alpha_o$. Consequently, the aggregate price index is only defined up to a constant

scaling factor. Prior to estimation, the crop share equation is dropped to ensure a non-singular covariance matrix and symmetry and the adding-up constraints are imposed on the model.

Estimates of the parameters of the three revenue share equations, derived using Zellner's seemingly unrelated regression (SUR) procedure, are reported in Table 1.[10] All own output price coefficients are positive and statistically significant at the 5 percent level. Furthermore, all remaining price coefficients are statistically significant at the 5 percent level.

Convexity conditions are checked by computing the eigenvalues of the Hessian matrix. These values were determined at the mean of the exogeneous variables and equalled .332, .102, and .001, indicating that convexity is satisfied.

Using the estimated coefficients in Table 1, measures of output supply and cross price elasticities can be calculated. These elasticities are derived holding total output constant and should be considered compensated or Hicksian supply elasticities. These estimates are presented in Table 2. All own output supply elasticities are positive and inelastic. For cattle and crops, the supply elasticities are measured at .623 and .889 respectively, whereas inventories had a supply elasticity of only .147. These elasticities indicate that producers respond relatively more to changes in current prices than to expected price changes.

In determining output substitution possibilities, recall that negative cross price elasticities imply substitutability and positive

TABLE 1

Regression Results: Translog Price Index

Share	Prices				Dummy Variables			
	Cattle	Inven*	Crops	Const.	1979	1980	1981	R^2
Cattle	.421	-.230	-.191	.595	-.056	.0001	.056	.513
	(5.4)**	(6.5)	(3.3)	(6.4)	(3.3)	(.006)	(3.6)	
Inven.	-.23	.325	-.095	.652	.023	.014	-.037	.556
	(6.5)	(6.6)	(2.2)	(10.0)	(1.0)	(.81)	(1.9)	
Crops	-.191	-.095	.286	-.247	.033	-.014	-.019	

* End-of-period inventories
** t-statistics in parentheses
(Five iterations were required for convergence)

TABLE 2

Output Supply and Cross Price Elasticities
Holding Total Output Constant, Mean of
the Exogeneous Variables, 1981

Quantity	Prices		
	Cattle	Inventories	Crops
Cattle	.623	-.200	-.424
Inventories	-.127	.147	-.020
Crops	-.827	-.062	.889

cross price elasticities imply complementarity. Using this definition, the estimated cross price elasticities indicate a substitute relationship amongst all output categories. For example, a one percent increase in cattle prices results in a decrease in inventories and crop output of -.127% and -.827%, respectively. On the other hand, a one percent increase in crop prices causes a small reduction in inventories (-.02%) but a somewhat larger shift away from cattle production (-.424). These results indicate that there is significant scope for changing the output composition between crops and cattle on cow-calf farms in western Canada.

The aggregate price index can now be calculated using the estimated coefficients from the share equations and, subsequently, the normalized profit function (Equation (6)) can be specified. (A random error term and yearly dummy variables are appended to the profit function prior to estimation.) In addition, by applying Hotelling's lemma to the normalized profit function, the derived net input demand equations can be defined as:

$$(10) \qquad q_j = -\beta_j \frac{\Pi^*}{W_j^*} \qquad j = 1, 2, 3,$$

where q_j is the jth input quantity.[11]

Efficient parameter estimation of the normalized profit function can be generated by simultaneously estimating Equation (6) and (10) with symmetry restrictions imposed (Yotopoulos, Lau and Lin). These equations are estimated using Zellner's SUR procedure and are exhibited in Table 3. All own input price coefficients are negative and statistically

significant at the 5 percent level, indicating that monotonicity and convexity are satisfied. In addition, all other parameters (except the dummy variable for 1979) are statistically significant at the 5 percent level.

It is interesting to examine the estimated coefficients for the dummy variables in Table 3. For the years 1980 and 1981, the estimated coefficients are statistically significant at the 5 percent level. These results provide evidence that the intercept term has shifted between the base year (1978) and 1980, and again shifted between the base year and 1981. This indicates that the data are not drawn from a homogeneous sample. Consequently, to generate efficient parameter estimates, it is necessary to maintain the dummy variable specification in the estimation.

The estimated coefficients in Table 3 can be used to compute measures of own price elasticities, cross price elasticities, and elasticities with respect to the fixed factor. These values are reported in Table 4.

With respect to own price, the aggregate output supply elasticity is positive and inelastic whereas all input demand elasticities are negative and elastic. This indicates that aggregate output is relatively inflexible to changes in aggregate output price whereas all factor inputs are altered substantially in response to own price changes. This latter effect is consistent with results reported by Yotopoulos, Lau and Lin. Furthermore, it is interesting to note that the own elasticity for materials is measured as being more responsive to own price changes than either labour or captial.

TABLE 3

Joint Estimation: Normalized Cobb-Douglas Profit Function and Net Input Demand Equations

Variable	Parameter	Estimated Coefficients*
Profit Function		
Constant	$\ell n\ A$	2.46 (7.4)
Labour	β_1	-.059 (16.8)
Capital	β_2	-.055 (16.8)
Materials	β_3	-.114 (12.6)
Beginning Inventories	γ	1.06 (39.28)
Dummy Variables		
1979 = 1		-.041 (.686)
1980 = 1		.151 (2.6)
1981 = 1		.237 (3.9)
Net Input Equations		
Labour	β_1	-.059 (13.3)
Capital	β_2	-.055 (16.8)
Materials	β_3	-.114 (12.6)

* t-statistics in parentheses

TABLE 4

Own Price Elasticities, Cross Price Elasticities, and Elasticities with Respect to the Fixed Factor: Cobb-Douglas Profit Function

Quantity	Price				
	Aggregate Output	Labour	Capital	Material	Beginning Inventories
Aggregate Output	.228	- .059	- .055	- .114	1.06
Labour	1.228	-1.059	- .055	- .114	1.06
Capital	1.228	- .059	-1.055	- .114	1.06
Material	1.228	- .059	- .055	-1.114	1.06

TABLE 5

Total Supply Elasticities: Output Component

Output	Prices		
	Cattle	Inventories	Crops
Cattle	.697	-.084	-.386
Inventories	-.049	.263	.018
Crops	-.753	.054	.927

The sign of the cross price elasticities defines inputs as substitutes or complements. On the input side, negative cross price elasticities imply complementarity and positive cross price elasticities imply substitutability. The results in Table 4 (as expected) indicate a complementary relationship between all input pairs.[12] Finally, the elasticities of aggregate output and factor inputs with respect to beginning inventories are estimated to be almost one indicating that a one percent increase in beginning inventories will result in a corresponding increase in factor demands and aggregate output.

To complete the presentation of the econometric results a recalculation of the output supply and corresponding cross price elasticities to account for the additional effect of changes in aggregate output on the supply of individual outputs is required (Fuss). These Marshallian elasticities are reported in Table 5 and indicate that total output supply elasticities are positive and inelastic but larger in magnitude than the corresponding compensated elasticities reported in Table 2. This is consistent with Le Chatelier Principle (Silberberg). The cross price elasticities still measure a substitute relationship between cattle and crops and between cattle and inventories. However, accounting for changes in aggregate output has altered the relationships between inventories and crops from substitutes to complements. A one percent increase in expected cattle prices results in an increase in crop supply of .054%. Similarly, a one percent increase in crop prices results in a small incrase in inventories of .018%. The former effect may be explained in that the resulting increase in crop supply is used in

maintaining larger inventories. However, the latter effect appears counter intuitive. One would expect that an increase in crop prices would increase the cost of feeding cattle and result in a reduction in inventories, ceteris paribus. One explanation may be that the measured relationship is capturing a time preference of farmers for shifting beef production from the current period into the future.

In summing up, the estimated functions provide reasonable estimates of elasticities of choice for both output and input variables and satisfy the restrictions imposed by economic theory. Such results may not have been achieved by estimating either a FFF or a more restrictive functional form specified to directly represent the multi-output, multi-input profit function.

Concluding Note

In this paper it has been shown that the C-D specification is inappropriate for modelling multi-output technologies. However, a procedure has been presented that enables the researcher to specify an FFF over output variables but yet maintain a C-D specification over input variables. This procedure has been used to estimate the structure of the cow-calf industry in western Canada.

From the estimation two conclusions can be drawn. First, cow-calf producers respond relatively more to changes in current prices than to expected future price changes. This implies that short term price fluctuations in the cattle market will result in significant

changes in current cattle supply. Second, there is significant scope for changing the output composition between cattle and crops on cow-calf farms. This suggests that during periods of falling cattle prices, cow-calf producers will shift away from cattle production and increase the production of crops.

Footnotes

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1. Statistics Canada, Farm Cash Receipts, Cat. No. 21-001, Ottawa, Queen's Printer, annual.

2. Statistics Canada, Selected Agricultural Statistics Canada and the Provinces, Ottawa, Queen's Printer, 1983.

3. The variable profit function is also defined by some authors as a gross or restricted profit function.

4. This failure may also be due to an inaccurate approximation of the true underlying function by the FFF or to an absence of optimizing behavior by producers.

5. The assumption of weak separability imposes the restriction that the output mix within the aggregate is independent of the level of the factor inputs.

6. The aggregator function $P_I(P)$ is assumed to be linear homogeneous in aggregate output. This assumption will ensure that the product of aggregate quantity and price will equal total revenue. See Diewert (1974) for a discussion of revenue functions.

7. A number of economic studies have attempted to model the economic options available to cow-calf producers. See Carvalho; Jarvis.

8. The FES does not provide direct information on cattle sold off farms. However, using a combination of inventory data, calving rates, and growth rates of animals in different categories, values for this variable were generated. End-of-period cattle inventories were obtained directly from the FES. And an index of crop output is calculated by dividing total receipts from the sale of crops, as reported in the FES, by an aggregate crop price index provided by Agriculture Canada. (It is necessary to include a total crop supply variable because, on examination, the data revealed that the beef farm, as defined, generated approximately 30 to 40 percent of total revenue from the sale of crops.)

9. The input indexes were calculated from expenditure data included in the FES. Labour expenditures included hired labour, family labour, and room and board. The hired labour wage rate is used as the labour price index and is assumed to be paid to both hired and family labour. Capital expenditures included repairs to buildings, repairs to fencing, capital depreciation, machinery expenses, taxes, custom work, financial loan expenses, land rental, and a flow variable representing the services from a given stock of land. The capital price index is generated using a Cobb-Douglas aggregator function and represents the rental price of annual per unit flows of services. Materials and services expenditures included feed and supplements, veterinary medicines, artificial insemination, telephone, electricity, fuel, irrigation, hardware, and

other operating expenses. Again, a Cobb-Douglas aggregator function is used to calculate a price index.

10. The estimated coefficients for the crop equation are calculated using the adding-up constraints.

11. The aggregate output supply equation (Q) can be derived as follows:

$$Q = \Pi^* - \sum_j \frac{\partial \Pi^*(.)}{\partial w_j^*} \; w_j^*.$$

12. For the C-D profit function, the cross price elasticity between inputs i and j is defined as $(\partial q_i/\partial w_j) \cdot (w_j/q_i) = \beta_j$. Monotonicity and convexity restrictions on the profit function require $\beta_j < 0$ for all j. Consequently, complementarity is imposed on all input pairs.

References

Almon, S., "The Distributed Lag Between Capital Appropriations and Expenditures", Econometrica, 33, 1965, 178-196.

Carvalho, J.L., "Production, Investment and Expectations: A Study of the United States Cattle Industry", Ph.D. dissertation, University of Chicago, 1972.

Diewert, W.E., "Walras' Theory of Capital Formation and the Existence of a Temporary Equilibrium", Discussion Paper No. 76, Department of Economics, University of British Columbia, 1972.

Diewert, W.E., "Functional Forms for Profit and Transformation Functions", Journal of Economic Theory, 6, 1973, 284-316.

Diewert, W.E., "Applications of Duality Theory", in Frontiers of Quantitative Economics, Vol. 2, (eds.) Intriligator, M.D. and D.A. Kendrick, Amsterdam, North-Holland, 1974.

Diewert, W.E., "Functional Forms for Revenue and Factor Requirements Functions", International Economic Review, Vol. 15, No. 1, 1974, 119-129.

Diewert, W.E. and T.J. Wales, "Flexible Functional Forms and Global Curvature Conditions", Discussion Paper 85-19, Department of Economics, University of British Columbia, 1985.

Fuss, M.A., "The Demand for Energy in Canadian Manufacturing", Journal of Econometrics, 5, 1977, 89-116.

Jarvis, L.S., "Cattle as Capital Goods and Ranchers as Portfolio Managers: An Application to the Argentine Cattle Sector", Journal

of Political Economy, 82, 1974, 489-520.

Judge, G.G., W.E. Griffiths, R.C. Hill, and T.C. Lee, The Theory and Practice of Econometrics, New York, John Wiley and Sons, 1980.

Lau, L.J., "Profit Functions of Technologies with Multiple Inputs and Outputs", The Review of Economics and Statistics, LIV, 1972, 281-289.

Lau, L.J., "A Characterization of the Normalized Restricted Profit Function", Journal of Economic Theory, 12, 1976, 131-163.

Lau, L.J. and P.A. Yotopoulos, "Profit, Supply and Factor Demand Functions", American Journal of Agricultural Economics, 54, 1972, 11-18.

Marshall, R.G., "Variations in Canadian Cattle Inventories and Marketing", Ontario Agricultural College, University of Guelph, 1964.

Silberberg, E.,The Structure of Economics: A Mathematical Analysis, New York, McGraw-Hill, 1978.

Yotopoulos, P.A., L.J. Lau and W.L. Lin, "Microeconomic Output Supply and Factor Demand Functions in the Agriculture of the Province of Taiwan", American Journal of Agricultural Economics, 58, 1976, 333-340.

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